Strategies to promote mathematical argumentation that builds content understandings

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Session Goals

Engage in tasks that illustrate how argumentation practices support the development of content understandings

Consider how specific tasks, structures & instructional strategies promote argumentation in 6-12 classrooms
Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. Students came up with the following possible reasons. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4?

A. Four is an even number, and odd numbers are not divisible by even numbers.
B. The number 100 is divisible by 4 (and also 1000, 10,000, etc.).
C. Every other even number is divisible by 4, for example, 24 and 28 but not 26.
D. It only works when the sum of the last two digits is an even number.
Why focus on argumentation?

Shifter, 1999

Mathematical reasoning is an evolving process of conjecturing, generalizing, investigating why, and developing and evaluating arguments.
SMP # 3 construct viable arguments and critique the reasoning of others -

understand and use stated assumptions, definitions, and previously established results in constructing arguments.

make conjectures and build a logical progression of statements to explore the truth of conjectures.

analyze situations by considering cases and counterexamples.

justify conclusions, communicate them to others, and respond to the arguments of others.

compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed.

Claim: these practices promote understanding
Does this always work?

Justify your decision

\[
\frac{4}{15} \div \frac{2}{3} = \frac{(4 \div 2)}{(15 \div 3)}
\]
Does this always work?

Justify your decision

\[ x^2 - 5x + 6 = 2 \]
\[ (x - 2)(x - 3) = 2 \]
\[ x - 2 = 2 \quad \text{or} \quad x - 3 = 2 \]
\[ x = 4 \quad \text{or} \quad x = 5 \]
Does this always work?

Justify your decision

\[3^{2x+1} + 3^2 = 3^{x+3} + 3^x\]
\[(2x + 1)2 = (x + 3)x\]
\[x^2 - x - 2 = 0\]
\[(x - 2)(x + 1) = 0\]
\[x = 2 \text{ or } x = -1\]
Where’s the math? --- Putting “does it always work” in action

- Does this always work directly addresses SMP #3
- Examining work forces us to think more carefully about why (procedures with connections)
- Using strategies and operations that you might not have considered can lead to noticing about structure (SMP #7: Look for and make use of structure)
- It encourages creative problem solving

Turn and talk - where might you use this type of activity in your own classroom
Which would you choose?

DUELING DISCOUNTS

Make an argument about which coupon is a better deal.
Which would you choose?

**Constructing an Argument**
- Explore the task individually
- Work with a small group to
  - Stake a claim and begin formulating your argument
- Create a group poster to communicate your argument

**Critiquing an Argument**
- Gallery walk
- Use the sticky notes to record affirmations - what about the argument convinced you or what connection did you make to your own argument?
- Use the sticky notes to write questions you have for the group.
Always, Sometimes, Never

a. A rhombus is a square
b. A triangle is a parallelogram
c. A square is a parallelogram
d. A square is a rhombus
e. A parallelogram is a rectangle
f. A trapezoid is a quadrilateral

The sum of 3 numbers is odd.

If you add 2 odd numbers you get an odd number.
How might students justify?

The graphs of two equations in a system of linear equations are shown.

How many solutions does the system have? Explain.
How might students justify?

The graphs of two equations in a system of linear equations are shown.

How many solutions does the system have? Explain.
What are the characteristics of tasks, structures, instructional strategies that invite students to create arguments?
Promoting Argumentation

Force students to make a choice
Provide a “foil” or provoke a disagreement
Make arguments publicly available for critique
  Convince yourself, a friend, a skeptic

Explicitly ask for justification
  Model “good justifications”
  Encourage multiple representations
How do you ensure the arguments elicit or deepen conceptual understanding?
Making sure arguments elicit or deepen conceptual understanding

Identify the key concepts / ideas you want to focus on - be specific

Anticipate the arguments that would provide evidence of understanding

Anticipate questions that will prompt students to add on to, clarify their arguments

E.g., what are your assumptions. Does this argument show why it “always” works? What is “special” about these numbers (sets of numbers)?

Choose an open task that has potential to elicit the math ideas on which you want to focus

ASK for justification

Model “good” arguments
What are your big takeaways?
What is one thing you think you might try -- or one thing you are still wondering about?