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Kathleen Jalalpour and Corrinne Lieu

BLOG: Follow a year in 5th grade, week by week.
https://singapore-math-blog.com/
or Google: “Off the Beaten Math”, the Pi Project

Keys Middle School, Palo Alto, CA (5th – 8th)
1/3 of our students were ending up the remedial math track.

The long-term consequences are fateful.

Tracking damages students who are tracked into the low track; it damages schools and ultimately, society as a whole.

Back to our college textbooks...
3 types of Learners

- **Visual/verbal learners**
  - Need time to think
  - Memorization is hard
  - Often good at seeing relationships, creative solutions

- **Abstract/Linear learners**
  - Fast
  - Memorize easily
  - Might struggle with written representation

- **Middle Group**
  - A mix

Therefore, All Our Work Has 3 Levels:

- **Level One: Concrete/Pictorial**
- **Level Two: Transfer to the Abstract**
- **Level Three: Challenge Problems**
GRADING

Examples of the 3 Levels
<table>
<thead>
<tr>
<th>Points</th>
<th>Score</th>
<th>Out of</th>
</tr>
</thead>
<tbody>
<tr>
<td>SGRs</td>
<td>32</td>
<td>34</td>
</tr>
<tr>
<td>HW</td>
<td>--</td>
<td>24</td>
</tr>
<tr>
<td>Quizzes, Level 1</td>
<td>--</td>
<td>18</td>
</tr>
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<td>Quizzes, Level 2</td>
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<tr>
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<td>0</td>
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<td>FINAL GRADE</td>
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Parent Signature/Comments:
### LEVEL 1
Making Meaning at the Concrete/Pictorial Level

<table>
<thead>
<tr>
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<th>Total you scored</th>
<th>Points:</th>
<th>Out of: maximum required for this level</th>
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<td>25</td>
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<tr>
<td>Quizzes Level1 (out of 17 possible)</td>
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<td>15</td>
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<tr>
<td>Quizzes Level2 (out of 44 possible)</td>
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<td><strong>Total</strong></td>
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**GRADE** 100%

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### LEVEL 2
At Grade Level (Transfer to the Abstract)

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</tr>
<tr>
<td>HW</td>
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<td>25</td>
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<td>Quizzes Level1 (out of 17 possible)</td>
<td>15</td>
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<td>15</td>
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<td>Quizzes Level2 (out of 44 possible)</td>
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<td>19</td>
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**GRADE** 99%

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### LEVEL 3
Optional Challenge

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<td>Quizzes Level1 (out of 17 possible)</td>
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**GRADE** 87%

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### LEVEL 1
Making Meaning at the Concrete/Pictorial Level

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**GRADE** 70%

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### LEVEL 2
At Grade Level (Transfer to the Abstract)

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**GRADE** 63%

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### LEVEL 3
Optional Challenge

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<td>7</td>
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**GRADE** 53%

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Making Meaning at the Concrete/Pictorial Level

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<td><strong>93</strong></td>
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**GRADE** 87% **B+**

### LEVEL 2
At Grade Level (Transfer to the Abstract)

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<td><strong>Total</strong></td>
<td><strong>91</strong></td>
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**GRADE** 82% **B-**

### LEVEL 3
Optional Challenge

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<td>17</td>
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<td>30 (out of 44 possible)</td>
<td>30</td>
<td>44</td>
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<tr>
<td>Quizzes Level3</td>
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<td><strong>Total</strong></td>
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**GRADE** 73%
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<th>LEVEL 3</th>
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<tr>
<td></td>
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<td>(maximum required for this</td>
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<tr>
<td>(17 possible)</td>
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<td>(4 possible)</td>
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<tr>
<td>(3 possible)</td>
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**GRADE**

100%  100%  A+  90%  A-
Every test/quiz has ALL 3 levels.

All students must do Levels 1&2. Level 3 is optional, if time.

Test/Quiz Breakdown: Although everyone does all three levels, they are graded at a different % on questions from the 3 levels.

**Level One: Concrete/Pictorial**
- Accessible to all students, since it is visual, not abstract.

**Level Two: Transfer to the Abstract**
- At grade level: more algorithms and more word problems

**Level Three: Challenge**
- For fastest workers
TEACHING

Each Grade Level has 2 or 3 “Non-negotiables”

• 5th: Fractions – at least pictorially

• 6th: Fractions, Signed numbers, solving equations (at least pictorially)

• 7th: Equations with fractions, signed numbers, percents, ratios

• 8th: Graphing Equations, systems of equations, monomials
In the Classroom

Concrete Pictorial Challenge

Level ONE, 5th Grade

What do you see?
Transferring to the Abstract

1a. \[
\begin{array}{c}
\text{ } \\
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\end{array}
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\text{ } \\
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\text{ } \\
\text{ } \\
\text{ } \\
\end{array}
\]

1b. \[
\begin{array}{c}
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
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\text{ } \\
\text{ } \\
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\text{ } \\
\text{ } \\
\text{ } \\
\end{array}
\] = \[
\begin{array}{c}
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\text{ } \\
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\text{ } \\
\text{ } \\
\text{ } \\
\end{array}
\]

2a. \[
\begin{array}{c}
\text{ } \\
\text{ } \\
\text{ } \\
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\text{ } \\
\text{ } \\
\end{array}
\] = \[
\begin{array}{c}
\text{ } \\
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\text{ } \\
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\text{ } \\
\text{ } \\
\end{array}
\]

2b. \[
\begin{array}{c}
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\end{array}
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\text{ } \\
\text{ } \\
\end{array}
\]

Fog Stone Isle
**WHAT ABOUT CHALLENGE?**

(i) A little monkey had 60 peaches.

On the first day he decided to keep $\frac{3}{4}$ of his peaches. He gave the rest away. Then he ate one.

On the second day he decided to keep $\frac{7}{11}$ of his peaches. He gave the rest away. Then he ate one.

On the third day he decided to keep $\frac{5}{9}$ of his peaches. He gave the rest away. Then he ate one.

On the fourth day he decided to keep $\frac{2}{7}$ of his peaches. He gave the rest away. Then he ate one.

On the fifth day he decided to keep $\frac{2}{3}$ of his peaches. He gave the rest away. Then he ate one.

**How many did he have left at the end?**

(ii) A little monkey had 75 peaches.

Each day, he kept a fraction of his peaches, gave the rest away, and then ate one. These are the fractions he decided to keep:

<table>
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<tr>
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<th>1</th>
<th>1</th>
<th>3</th>
<th>3</th>
<th>5</th>
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<td>4</td>
<td>5</td>
<td>6</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

In which order did he use the fractions so that he was left with just one peach at the end?

(iii) Whenever the monkey has some peaches, he always keeps a fraction of them each day, gives the rest away, and then eats one. I wonder how long he could make his peaches last for...

Here are his rules:

- Each fraction must be in its simplest form and must be less than 1.
- The denominator is never the same as the number of peaches left.
  For example, if there were 45 peaches left, he would not choose to keep $\frac{45}{45}$ of them.

**Can you start with fewer than 100 peaches and choose fractions so that there is at least one peach left after a week?**

**Starting with fewer than 100, what is the longest you can make the peaches last?**
Level ONE

Solve by drawing only. No equations. Count and answer on line.

\[
\frac{3}{5} + 1\frac{7}{10}
\]

Level ONE

LEVEL 1 - Fractions

1. Drawing a model is required on this problem. (2 points)

\[
\frac{1}{4} \cdot 2 - \frac{7}{8}
\]

Draw \(\frac{1}{4}\) TWICE (in 8ths).

Then subtract \(\frac{7}{8}\) by crossing out, then count up the \(\frac{8}{8}\)th.
Subtraction?

Level ONE

4. Solve in 2 ways:

\[ \frac{2\frac{1}{2}}{2} - \frac{1\frac{2}{3}}{3} = \frac{5}{6} \]

a. By drawing:

b. By equations using common denominators:

Transfer to the Abstract

Level TWO

1. Fractions. Try to do these with mental math; otherwise, always feel free to use a drawing! Simplify.

a) \[ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \]

b) \[ \frac{3}{5} + \frac{1}{2} + \frac{7}{10} = \]

Find the sum of all 3 fractions:

\[ \frac{4}{5} \]

\[ \frac{1}{3} \]

\[ \frac{1}{6} \]
2. Sally and Cindy have 63 balloons. \( \frac{2}{5} \) of Sally’s balloons is equal to \( \frac{1}{3} \) of Cindy’s balloons. How many balloons does Sally have?

4) 12 of the children in class A were girls. There were 10 boys in the class. Class B had twice as many girls as class A and half as many boys as class A. When the 2 classes are put together, how many MORE girls than boys are there in total?
Two 600 ml pitchers contain vinegar. One pitcher is $\frac{1}{3}$ full and the other pitcher is $\frac{2}{5}$ full. Oil is added to fill each pitcher completely, and then both pitchers are poured into one large container.

What fraction of the mixture in the large container is vinegar?
Level ONE, 6th Grade

(c) $3x + 7 = 2x + 17$

b. $\frac{5}{6} + \frac{1}{2}x = 1$

= $\frac{1}{2}$ of what?

1
b. \(\frac{5}{6} + \frac{1}{2}x = 1\)
Level ONE
6th / 7th Grade

\[2(x + 3) - \frac{1}{2}x = 9\]

\[\frac{1}{2}x + 6 = 9\]

\[\frac{1}{2}x = 3\]

\[\frac{1}{2}x = 1\]

\[x = 2\]
CHALLENGE

Critical thinking Company

Books and eBooks
After an increase of 12%, Wyatt’s new salary was $8.90 per hour. What was his old salary? Use a calculator, and round to the nearest penny.

**Hint:**
c) Kyle spent 15% of his allowance money on a football. If the football cost $60, how much money did he start with?

\[
60 = 0.15 \times \text{what?}
\]

OR....

\[
60 = 0.15 \times x
\]

....

---

**Level ONE, 6th Grade**

The ratio of a steel cable to a rope is 8:3.

The total length of the two, end-to-end, is 176 m.

How long is the rope?
Mr. Lewis is keeping fowl in his garage (Mr. Lewis’ middle name is Fowler, after all). The ratio of chickens to ducks is 1 : 3, and the ratio of ducks to geese is 2 : 7. If he has 24 chickens, how many total fowl does he have?

Suppose \( a \) represents the same negative number in each expression below. Order the expressions from least to greatest.

\[
\begin{align*}
    a - 5a + 11a & \quad -6a + 1 + 3a + 3 & \quad 2a + 12a - 1
\end{align*}
\]

We have \( \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \) and \( \frac{1}{3} + \frac{1}{4} = \frac{7}{12} \).

If we continue this pattern, computing \( \frac{1}{4} + \frac{1}{5}, \frac{1}{5} + \frac{1}{6} \), and so on, what is our first result that is smaller than \( \frac{1}{5} \)?
Without using a calculator, determine which is larger: $5\sqrt{13}$ or $3\sqrt{23}$.

The length and width of a rectangle are each increased by 10%. By how much do the perimeter and area change, respectively?

In the figure shown, points $D$, $E$, and $F$ are the midpoints of semicircles $ADB$, $BEC$, and $DFE$, respectively. If the radius of each semicircle is 1, then what is the area of the shaded region? Source: MATHCOUNTS

8th Grade
Graph the coordinates. Then find the slope and the y-intercept.

\[
\begin{array}{|c|c|}
\hline
x & y \\
-2 & 3 \\
0 & 1 \\
2 & -1 \\
4 & -3 \\
\hline
\end{array}
\]

\[
\frac{8 + 2x}{3} = 40
\]
2. Graph by any method:
   \[ y = 3x - 2 \]
   \[ y = -\frac{1}{2}x + 4 \]
Bucharest have contributed to the edited solution below.
As triangle DEF is equilateral, all its angles are 60°.
Angle AEF = 180° - 60° - c = 120° - c
Similarly
Angle BFD = 120° - b
Angle EDC = 120° - a
From triangle FAE, I calculate angle A:
As triangle ABC is an isosceles Angle ABC = Angle ACB = x
Therefore 180° - (120° - b + a) = 180° - (120° - a + c)
Therefore 60° + b - a = 60° + a - c
Therefore 2a = b + c
Therefore a = (b + c)/2
Part Two
If a = b = c
Angle ABC = 180° - (120° - b + a) = 180° - (120° - a + a) = 60°
Angle ACB = 180° - (120° - a + c) = 180° - (120° - a + a) = 60°
Therefore angle BAC is 60°
Therefore triangle ABC is equilateral.

Where to find Challenge

Math in Focus
NCTM Illuminations
desmos
NRICH nrich.maths.org
AoPS The home of rich mathematics.
Math Pickle
BRILLIANT

Enrichment books, Math in Focus
VERDICT

• Improvement in achievement for our most vulnerable students.
• Greater ownership of their own work.
• Improved preparedness for algebra
• Minimal parent opposition because of the availability of challenge.

THANK YOU!

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